

## Practical Local Geoid Model Determination for Mean Sea Level Heights of Surveys and Stable Building Projects

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**Abstract:** A local geoid model for Eboriaria, Benin City using the geometric (GPS/Levelling) method was determined for calculation of mean sea level heights. Fifty points were established for the model and ten points were used for interpolation. The geoid heights were determined by finding the difference between the observed orthometric heights and the ellipsoidal heights. The polynomial regression model  $D$  was used for the interpolation of the orthometric heights. The computed mean standard deviation between the observed orthometric heights and the interpolated orthometric heights was  $\pm 21\text{cm}$ . A mean geoidal undulation of  $28.410\text{m}$  was computed using the gravimetric method. The computed orthometric heights using the gravimetry mean geoidal undulation were compared with the observed orthometric heights and seen to be identical. It is recommended that orthometric heights of project areas should be determined from GPS observations with the local geoid model of the area also determined.

**Keywords:** Geoid Model, Ellipsoidal Height, Orthometric Heights, Geoid Undulation, GPS

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Date of Submission: 29-05-2018

Date of acceptance: 12-06-2018

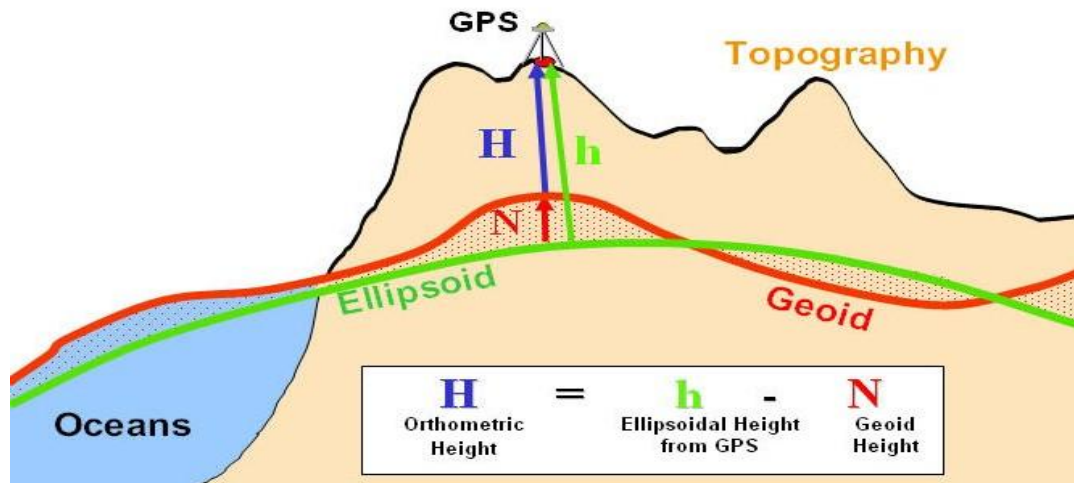
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### I Introduction

One of the major tasks of geodesy is the determination of the geoid which can be defined as the equipotential surface of the gravity field of the earth. Nowadays, this task is getting even more crucial due to the development of Global Positioning System (GPS). This system gives three dimensional positioning anywhere on the earth, but in the field of engineering the system is inadequately used only for two dimensional positioning. This is due to the fact that global positioning system gives ellipsoidal heights which are geometric heights unlike orthometric heights which have more physical meaning.

However, most surveying measurements are made in relation to the geoid which is the equipotential surface of the earth gravity and not the ellipsoid because the instrument is aligned with the local gravity vector which is perpendicular to the geoid surface, normality through the use of a spirit bubble (Featherstone, et al., 1998). The geoid height or the geoidal undulation ( $N$ ) is described as the separation of the geoid from the ellipsoid of revolution (Ahmed, and Derek, 2011).

Consequently, ellipsoidal heights cannot satisfy the aims in practical surveying, engineering or geophysical applications as they have no physical meaning and must be transformed to orthometric heights ( $H$ ) which are referred to geoid to serve the geodetic and surveying applications (see Fig. 1). To accomplish this transformation between the ellipsoid heights and orthometric heights, the geoidal undulation ( $N$ ) from the ellipsoid must be known. Basically a WGS 84 ellipsoidal height ( $h$ ) is transformed to an orthometric height ( $H$ ) by subtracting the geoid – WGS 84 – ellipsoid separation ( $N$ ) which is called the geoid undulation (Erol and Celik, 2004).



**Fig. 1: Relationship between orthometric, geoid and ellipsoidal heights**

Source: Ono (2013).

Depending on data availability and accuracy requirements, there are many principle approaches for determining Geoid models; some of the approaches are gravimetric method, geometric method and the astrogeodetic method (Ono, 2009 and Okeke and Nnam, 2016). The geometric method involves the use of GPS and levelling data, where both the ellipsoidal and orthometric heights are given. A mathematical relation depicting the surface of the geoid with regard to the reference ellipsoid is known as geoid model, equation (1) (Rabindra et al, 2017).

$$N = h - H \quad (1)$$

The gravimetric method can be carried out by solution of the well-known Stokes – integral, (Heiskanen and Moritz 1967).

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g S(\psi) d\sigma \quad (2)$$

where,  $N$  geoid undulation,  $\Delta g$  – mean gravity anomaly,  $S(\psi)$  - Stokes function and  $\gamma$  - normal gravity of the earth.

The aim of this study is to determine a local geoid model for mean sea level heights of surveys and building projects with a view of ensuring stable projects constructions using GPS /Levelling (Geometric) method. The Primary objectives are:

- i. To determine the geoid undulation of each of the selected points within the study area.
- ii. To produce the observed, EGM08 and EGM96 orthometric heights and the geoid undulations contour maps of the study area.
- iii. To determine a mean geoidal undulation using the gravimetric method
- iv. To compute orthometric heights using the determined gravimetric mean geoidal undulation and comparing the computed orthometric heights with the observed orthometric heights.

The study area is Evboriaria in Benin City. It is a developing area located along Sapele Road in Ikpoba Okha Local Government Area of Edo State. The area lies between latitudes  $06^{\circ} 15' N$  and  $06^{\circ} 18' N$  and longitudes  $05^{\circ} 36' E$  and  $05^{\circ} 39' E$ . It occupies an area of about 1.6 square kilometers with a population of about 2,200 according to 2006 National Population Census.

This study was limited to the determination of the local geoid model of Evboriaria in Benin City. The scope of the study is as follows:

- i. GPS observations of chosen Points.
- ii. Determination of Orthometric heights of the selected points using spirit levelling and plotting of the heights contours
- iii. Computation of the geoidal undulation of each point and plotting of the geoid contour of the study area.
- iv. Computation of the EGM08 and EGM96 geoidal undulations and orthometric heights of the points within the study area and plotting of their orthometric heights contours.
- v. Determination of a mean geoidal undulation using the gravimetric method and computation of orthometric heights using the determined mean geoidal undulation

### Polynomial Regression Modal D for interpolation

According to Isioye and Youngu (2009), this approach considers the use of a correcting term to be determined for an area or a project site with common stations with ellipsoidal and orthometric height (Co-located points) using the means of the difference between the global earth model and GPS/Levelling geoid heights, thus we have:

$$H_{(x)} = h - N_{EGM96} + \delta N \quad (3)$$

Where  $\delta N$  = corrective term =  $\Sigma(N_{\text{residual}})/n$ .

$n$  = number of stations

Equation (3) implies that for every new point to be determined a corrective term is added to the difference between the ellipsoidal and geoid height from EGM96 for the point.

## II Methodology

The methodology involved the following stages namely: data requirements, data acquisition, data processing, and data presentation.

### 2.1 Method of Data Acquisition

#### Reconnaissance

The study area was visited and suitable points were chosen and marked with wooden pegs at 100m intervals along the road. A nearby control was located and its coordinates were obtained from the Ministry of Lands and Surveys, Benin City.

Prior to the levelling measurement, the digital level (Sokkia SDL50) was tested using the two-peg-test so as to ensure that it was in good condition.

#### Monumentation

Prior to the spirit levelling and the GPS observation, Pre-cast Property Beacons (Survey Pillars) with dimensions 18cm x 18cm x 75cm (SURCON, 2003) were used to replace the wooden pegs that were fixed at the selected points during reconnaissance. The pre-cast property beacons consist of a good proportion of cements, gravel, sand and water mixed in a ratio of 1:2:3 respectively. A number (SEO) template was engraved on each of the beacons.

#### Procedure for Spirit Levelling

The levelling was carried out in three loops, the first loop started from FGPEY 33 which was located inside FERMA premises along Sapele road; through SEO 50, SEO 01 to SEO 27, SEO 48 and SEO 49 and closed on FGPEY 33. The second loop started from SEO 27 through SEO 28 to SEO 39 and closed on SEO 50. The third loop started from SEO 50 through SEO 40 to SEO 46 and closed on SEO 48 (see Fig. 2).

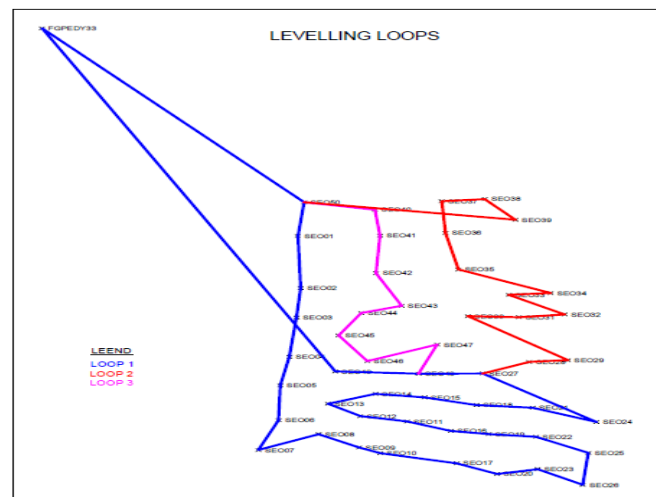


Fig. 2 The three Leveling Loops

#### Procedure for Differential GPS Observation

The GPS observations of the chosen points were carried out in five different days. The observation was used to obtain the three dimensional coordinates of the chosen points. The Base Receiver (Fig. 3) was set at the control station (FGPEY 33) that was located in FERMA premises along Sapele Road while the Rover Receiver was moving from one of the monument points to another during the observation. The post-processing static mode of operation was used for the observation.



**Fig.3: Base Receiver at Control**

Station FGPEY 33

The following were carried out each time the CHC900 (base or rover) receiver was set at a station:

1. The CHC900 was switched on.
2. The status switch was pressed down until the yellow light came up; this was done so as to set the receiver on static operational mode.
3. The station ID, instrument height, observation start time and stop time were recorded. The instrument heights and station IDs were used during downloading of the acquired data.
4. The receiver was switched off at the end of each observation (session) to preserve the battery life.

## **2.2 Data Processing Procedure**

### **Processing of Raw Data**

The reduced (orthometric) heights determined from the spirit levelling using the digital level were input into a computer system in a Microsoft Excel 2010 spreadsheet in the office. The GPS data were downloaded into a computer system by direct USB cable connection from the DGPS receiver to the computer system using HcLoader software. The downloaded data were processed and adjusted with Compass software.

### **Deduction of the EGM08 and EGM96 Geoid Undulations**

The observed geodetic coordinates of the stations were used online to compute the EGM08 and EGM96 geoid heights of the stations. The EGM08 and EGM96 geoid heights were calculated using UNAVCO and GeoidEval software respectively.

### **Computation of the Geoid Undulations and the EGM08 and EGM96 Orthometric Heights**

The transformed coordinates were copied into Microsoft Excel 2010 spreadsheet for the computation of the geoidal undulations. In the spreadsheet, the difference between the orthometric height and the ellipsoidal height of each point was computed to obtain the geoidal undulation, equation (1). The EGM08 and EGM96 orthometric heights were also computed in the Microsoft Excel spreadsheet.

### **Interpolation of the Orthometric Heights Using the Polynomial Regression Model D**

The polynomial regression interpolation Model D (Isioye and Youngu, 2009) was used for ten (10) randomly selected points. The interpolation of the orthometric heights of the selected points was also carried out in the Excel spread sheet where the local geoid heights, EGM08 orthometric heights and the EGM96 orthometric heights were computed. The standard error of the interpolated heights was also computed for each of the global geopotential models and a mean standard error was obtained.

### **Computation of the Mean Geoidal Undulations of Some Randomly Selected Points Using the Gravimetric Method**

The mean geoidal undulation of 15 randomly selected points was computed using the gravimetry method. The gravity of each station and its anomaly were computed with PTB software (online local gravity calculator). The mean geoidal undulation was computed using the mean gravity anomalies of the selected points. The computation of the mean geoidal heights using the gravimetric method was carried out in excel spread sheet. The gravimetric mean geoidal undulation was used to compute the orthometric heights of the selected points.

## 2.3 Results Analysis

### Analysis of the Spirit Levelling Result

The observed orthometric heights were seen to be in good shape as shown by the difference between the observed and the known heights of the closing stations. The closing error for the first loop was 0.0002m, the second loop closing error was found to be 0.0004m and that of the third loop was -0.0002m which were less than 1mm standard. The results were accepted as the closing errors of the three loops were each less than 1mm. Table 1 shows the results of the closing errors. The high accuracy of the levelling was as a result of the fairly flat topography of the study area, the observer's know-how and equipment used.

**Table 1: Known and Observed Height of the Closing Stations**

Station	Description	H <sub>(known)</sub> (m)	H <sub>(observed)</sub> (m)	ΔH (m)
FGPEDY 33	Starting and closing station	43.486	43.4862	0.0002
SEO50	closing station	30.307	30.3074	0.0004
SEO48	closing station	26.211	26.2108	-0.0002

### Analysis of the DGPS Result

The DGPS observations were carried out in five days. The observations of each day were processed as a separate loop. From the processing results, the five loops observations have passed the network adjustment test. That is, the normal matrix generated was a regular one and inverted accordingly for calculation of residuals. The undulations shown in Fig. 4 were as a result of the variation in the geoidal heights indicating that the geoidal undulation changed by no constant value.

### Interpolated orthometric heights and their Standard Error

The interpolation of the orthometric heights was carried out using ten randomly selected points in excel spread sheet (Table 2). The average residual between the local geoid model and the two global geoid models (EGM08 and EGM96) geoid heights of the points are 8.888 and 8.839 respectively. The standard error of the interpolated orthometric heights was computed to be  $\pm 0.21$ m. This means that orthometric heights can be computed within the study area with an accuracy of  $\pm 21$ cm.

**Table 2: Interpolated orthometric heights and their Standard Error**

Station	H <sub>(Observed)</sub> (m)	H <sub>EGM08</sub> (Polynomial Regression Model) (H <sub>EGM08</sub> - Σ(N <sub>LOCAL</sub> )/N) (m)	H <sub>EGM96</sub> (Polynomial Regression Model) (H <sub>EGM96</sub> - Σ(N <sub>LOCAL</sub> )/N) (m)	A (H <sub>(observed)</sub> - H <sub>(EGM08)</sub> ) (m)	B (H <sub>(observed)</sub> - H <sub>(EGM96)</sub> ) (m)	A <sup>2</sup>	B <sup>2</sup>
FGPEDY33	34.628	34.598	34.608	0.030	0.020	0.000900	0.000400
SEO02	29.589	29.757	29.759	-0.168	-0.170	0.028224	0.028900
SEO05	26.597	26.808	26.808	-0.211	-0.211	0.044521	0.044521
SEO11	24.501	24.481	24.48	0.020	0.021	0.000400	0.000441
SEO17	23.292	23.342	23.34	-0.050	-0.048	0.002500	0.002304
SEO26	28.245	28.361	28.358	-0.116	-0.113	0.013456	0.012769
SEO33	31.407	31.582	31.584	-0.175	-0.177	0.030625	0.031329
SEO39	34.731	34.196	34.198	0.535	0.533	0.286225	0.284089
SEO42	30.095	30.319	30.322	-0.224	-0.227	0.050176	0.051529
SEO48	26.211	26.326	26.326	-0.115	-0.115	0.013225	0.013225
$\sum(H_{(observed)} - H_{(EGM08)})^2/n (\sigma) =$						<b>0.047025</b>	<b>0.046951</b>

Average standard error ( $\sigma$ ) =  $\sqrt{(\sum (H_{(observed)} - H_{(EGM08)})^2/n)} = \pm \sqrt{0.046988} = \pm 0.21$ m

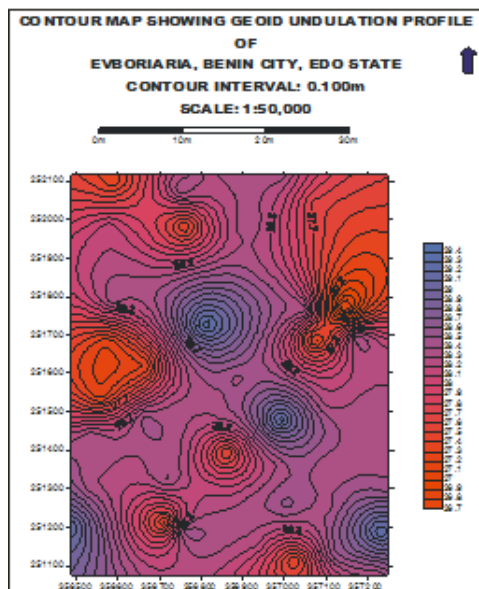
### Comparison between the Observed Orthometric Heights of the Selected Points and the Computed Orthometric Heights Using the Gravimetric Mean Geoidal Undulation

The mean geoidal undulation of 15 randomly selected points was determined using the gravimetric method (Table 3). The determined mean geoidal undulation was used to compute the orthometric heights of the selected points. It was observed from Table 3 that the orthometric heights computed with the gravimetric mean geoidal undulation were identical with the observed orthometric heights. It can also be seen from Table 3 that the mean change in orthometric height ( $\Delta H$ ) is 0.081m. The agreement of these results has demonstrated the GPS/Levelling (Geometric) method potential for local geoid determination in a small area.

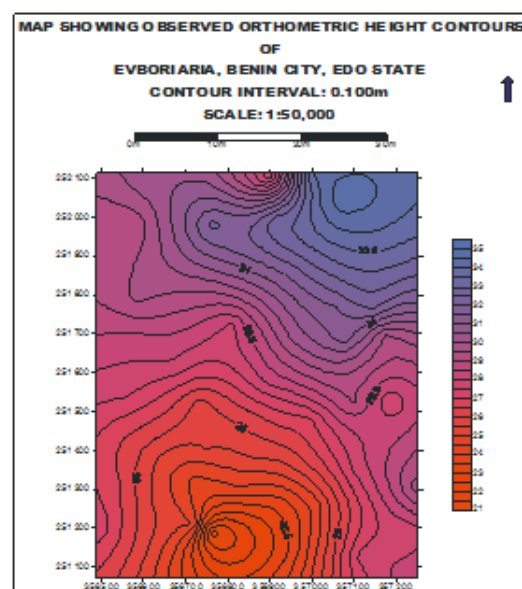
**Table 3: Comparison between the Observed Orthometric Heights of the Selected Points and the Computed Orthometric Heights Using the Gravimetric Mean Geoidal Undulation**

Gravimetric mean geoidal undulation = 28.410m				
Station	Ellipsoidal Height (h)	Observed Orthometric Height ( $H_{(observed)}$ )	Computed Orthometric Height ( $H_{(Computed)}$ (h-28.410))	Difference
SEO02	57.952	29.589	29.542	0.047
SEO05	54.988	26.597	26.578	0.019
SEO08	53.573	25.451	25.163	0.288
SEO11	52.654	24.501	24.244	0.257
SEO17	51.511	23.292	23.101	0.191
SEO26	56.525	28.245	28.115	0.130
SEO27	56.188	26.911	27.778	-0.867
SEO33	59.777	31.407	31.367	0.040
SEO36	61.164	32.767	32.754	0.013
SEO39	62.402	34.731	33.992	0.739
SEO42	58.519	30.095	30.109	-0.014
SEO43	57.326	27.939	28.916	-0.977
SEO46	54.160	25.862	25.750	0.112
SEO48	54.510	26.211	26.100	0.111
SEO50	57.598	30.307	29.188	1.119
Mean $\Delta H$ =				<b>0.081</b>

The contours of the observed, EGM08 and EGM96 orthometric heights were plotted with Surfer 8 software. From the plotted contours, it was observed that the orthometric height were identical to some extent. The resemblance is more evident between 251100mN – 251400mN and 356500mE – 357200mE, also between 251700mN – 252100mN and 356500mE – 356700mE. As seen in Figures 4 to 7.



**Fig. 4:** Geoidal Undulation Contour Map of the Study Area



**Fig.5:** Observed Orthometric Heights



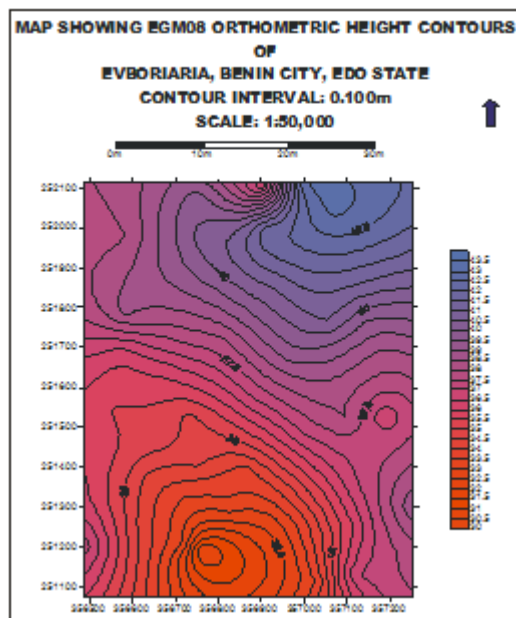


Fig. 6: EGM2008 Derived Orthometric Heights

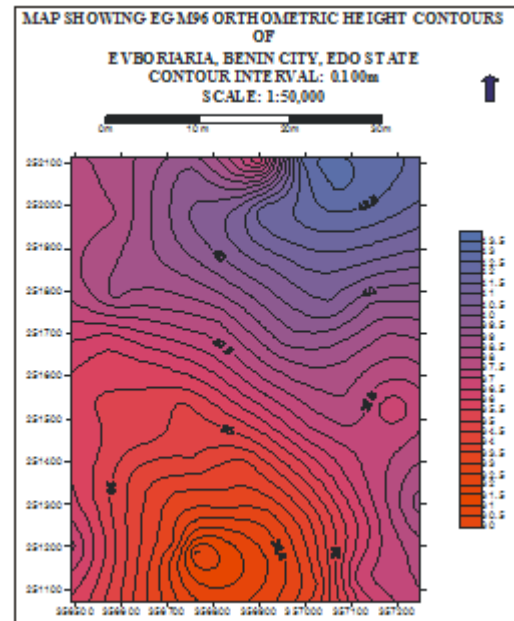


Fig. 7: EGM96 Derived Orthometric Heights

### III Conclusion

Vanicek (2009), concluded that if the engineering project is designed by means of properly defined heights including a properly defined datum (geoid) we can be assured that there will not be any nasty surprises, water run down the hill, a lake surface will have the same height everywhere etc. so, the geoid model of the study area was determined using the geometric method; fifty points were used altogether. The geoid heights were computed by finding the difference between the ellipsoidal heights and the orthometric heights of the points. The mean residuals obtained from the difference between the determined local geoid model and the EGM08 and EGM96 models were used for the interpolation of the orthometric heights using the Polynomial Regression Model D (Isioye and Youngu, 2009). Also, the standard error of the interpolated orthometric heights was computed.

The contour maps of the determined local geoid, the observed orthometric heights, the EGM08 derived orthometric heights and the EGM96 derived orthometric heights were plotted with Surfer 8 software. The orthometric heights contours were compared and seen to be identical. The computed orthometric heights using the gravimetric mean geoidal undulation were compared with the observed orthometric heights and seen to be identical.

### IV Recommendations

Based on the analysis of the geoid model determined in this study, the following recommendations are therefore made:

1. That the orthometric heights of any project area should be determined from GPS observations if the local geoid model of that area has already been determined. The use of assumed heights in project areas should be totally discarded as they have no relationship with the geoid or the physical surface. Not accounting for it might have contributed to project failures.
2. That the geometric method should be used for applications like engineering feasibility studies (earthwork determination), GIS topographic mapping, remote sensing land-cover mapping project and oceanographic application that required adequate and accurate height information.
3. Since the geometric method is relatively accurate, cheaper and faster than the traditional levelling technique or none of it, construction companies and survey firms should adopt it for the determination of practical heights which have relationship with the mean sea level in project areas for stability of projects.

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Eteje, S. O " Practical Local Geoid Model Determination for Mean Sea Level Heights of Surveys and Stable Building Projects." IOSR Journal of Environmental Science, Toxicology and Food Technology (IOSR-JESTFT) 12.6 (2018): 30-37.